Lessons 3-1, 3-2, 5-3 Relating the Graphs of Part 1

AP Calculus AB — Name *fkml 2016*

Lessons 3-1, 3-2, 5-3 *Relating the Graphs of* f, f', and f'' *Part 1* — Date

Learning Goals:

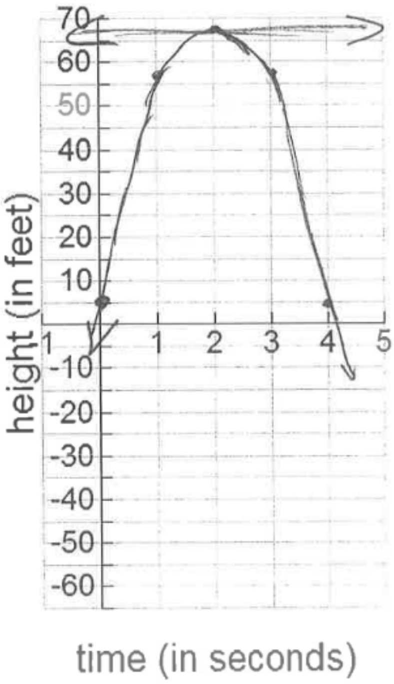
- *I can graph and analyze* f *from the graph of* f', *graph and analyze* f' *from* f, *and graph the derivative of a function given numerically with data.*
- *I can approximate derivatives graphically.*

Part 1 – The Graphs of f and f' - A Simple Example:

The height (in feet) over time (in seconds) of a projectile launched straight up into the air at 64 feet per second, from an initial height of 5 feet can be modeled by the equation $f(t) = 5 + 64t - 16t^2$.

1. Fill in the table below and plot points to sketch a graph of the function *f*. *[Note that since* $-\frac{b}{2a} = 2$, *the maximum value of the graph occurs at* $t = 2$]

t	0	1	2	3	4
$f(t)$	5	53	69	53	5



2. On the graph of f, sketch the tangent line to f at $x = 2$. What is the slope of this tangent line? $m = 0$

3. Fill in the below table for $f'(t)$, the derivative function. Plot points and sketch a graph of the derivative function on the same axis.

$f'(t) = 64 - 32t$

t	0	1	2	3	4
$f'(t)$	64	32	0	-32	-64

4. What is the value of $f'(2)$? How does the value of $f'(2)$ compare to your answer to question (2)? Why does this answer make sense?

$f'(2) = 0 \Rightarrow$ Same as slope of T.L. $x = 2$

☆ Derivative is the slope of T.L., so if derivative $= 0$, then T.L. should be horizontal.

5. What do all *y*-coordinates on the graph of f' represent in terms of the graph of *f*?

All y-coordinates of f' are slopes of T.L. of f at that x-value.

6. The derivative function f' should be linear. What does this fact mean about the rate of change of the function *f*?

The rate of change of f is decreasing at a constant rate.

OVER →

Lessons 3-1, 3-2, 5-3 Relating the Graphs of Part 1

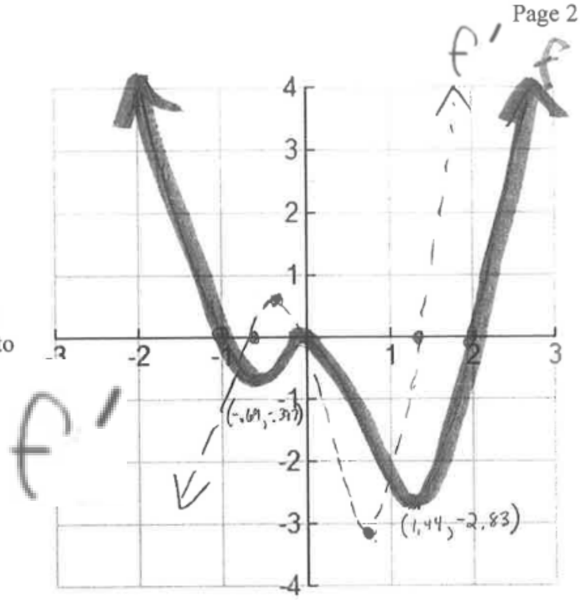
Part 2 – The Graphs of f and f' - In General

1. Let $f(x) = x^4 - x^3 - 2x^2$. f

   Find $f'(x)$.

   $f'(x) = 4x^3 - 3x^2 - 4x$

   Use your calculator to find all zeros and local extrema (that means local max/min) in order to plot BOTH graphs as accurately as possible [*remember, being able to find zeros and extrema on your calculator is necessary for the AP Exam!*]



2. Answer the following questions by inspection of the graphs:

   a. Over what intervals does the graph of f appear to be increasing (i.e. rising as you move from left to right)

   $(-.69, 0) \cup (1.44, \infty)$

   b. Over what intervals does the graph of f' appear to be positive (i.e. above the x-axis)?

   $(-.69, 0) \cup (1.44, \infty)$

   c. Over what intervals does the graph of f appear to be decreasing (i.e. falling as you move from left to right)

   $(-\infty, -.69) \cup (0, 1.44)$

   d. Over what intervals does the graph of f' appear to be negative (i.e. below the x-axis)?

   $(-\infty, -.69) \cup (0, 1.44)$

   e. What are the x-coordinates of all relative extrema of f (i.e. local maximum and minimums)?

   $x = -.69 \quad x = 0 \quad x = 1.44$

   f. For what values of x does the graph of f' appear to have zeros (i.e. x-intercepts)?

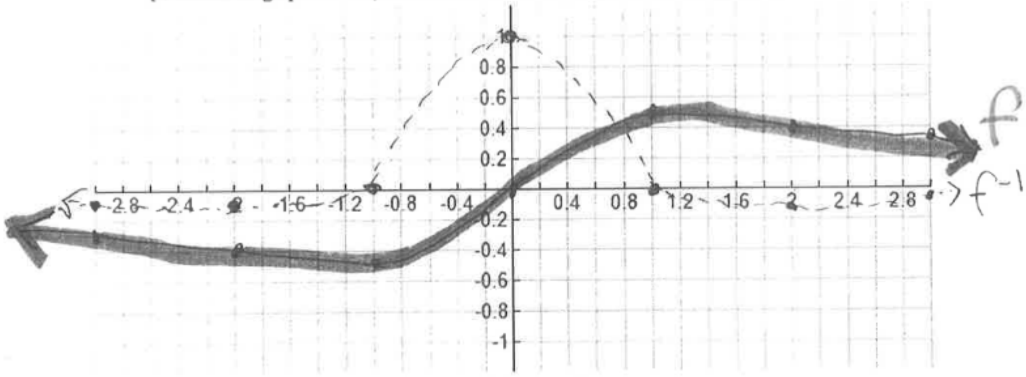
   $x = -.69 \quad x = 0 \quad x = 1.44$

Lessons 3-1, 3-2, 5-3 Relating the Graphs of Part 1

In calculator

3. Let $f(x)=\frac{x}{1+x^2}$. Graph both $f(x)$ and $f'(x)$ below. After entering $f1(x)=\frac{x}{1+x^2}$, to graph $f'(x)$, enter $f2(x)=\frac{d}{dx}(f1(x))$.

[to sketch the graphs below, make tables and find zeros, max/mins on your calculator]



4. Answer the following questions by inspection of the graphs:

a. Over what intervals does the graph of f appear to be increasing (i.e. rising as you move from left to right)? ↑

$(-1, 1)$

b. Over what intervals does the graph of f' appear to be positive (i.e. above the x-axis)?

$(-1, 1)$

c. Over what intervals does the graph of f appear to be decreasing (i.e. falling as you move from left to right)? ↓

$(-\infty, -1) \cup (1, \infty)$

d. Over what intervals does the graph of f' appear to be negative (i.e. below the x-axis)?

$(-\infty, -1) \cup (1, \infty)$

e. What are the x-coordinates of all relative extrema of f (i.e. local maximums and minimums)?

$x=-1$ min $\quad x=1$ Max

f. For what values of x does the graph of f' appear to have zeros (i.e. x-intercepts)?

$x=-1 \quad x=1$

g. Complete the table below:

x	-3	-2	-1	-0.6	0	0.6	1	2	3
f is inc., dec., or neither	Dec	Dec	Neither	Inc	Inc	Inc	Neither	Dec	Dec
f' is pos., neg., or zero	−	−	0	+	+	+	0	−	−

↘ Min ↗ $\quad$ ↗ Max ↘

− → + $\quad$ + → −

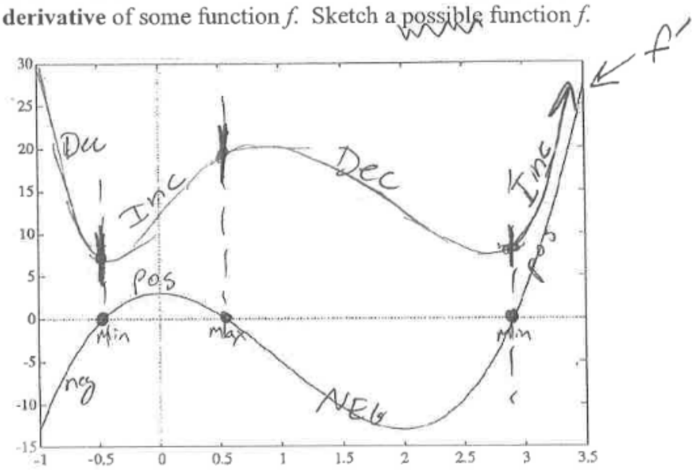
OVER →

Lessons 3-1, 3-2, 5-3 Relating the Graphs of Part 1

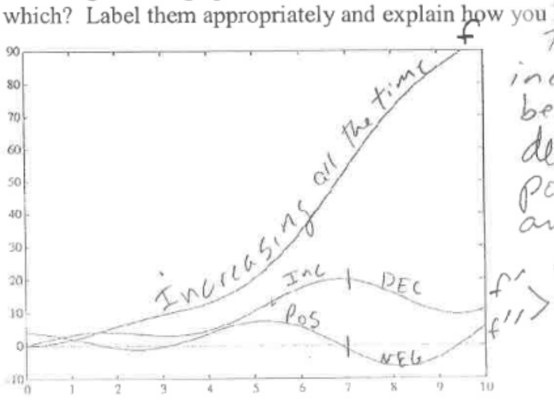
5. On the basis of your answers to problems 2 through 4, how are the graphs of any function f and its derivative f'related? Be as specific as possible.
[*You should have at least 3 statements*]

(1) When f is increasing, f' > 0. [deriv. is +]
(2) When f is decreasing, f' < 0 [deriv. is −]
(3) When f is at local extrema, f' = 0 [deriv. is 0]

6. So far in this investigation we have looked at the shape of the graph of f to help visualize the shape of its derivative function f'. What about going backwards? Suppose you have a graph of f', would you be able to visualize the graph of f? The graph below is a sketch of a **derivative** of some function f. Sketch a possible function f.



7. To the right is the graph of a function f, its first derivative, and its second derivative. Which graph is which? Label them appropriately and explain how you know.

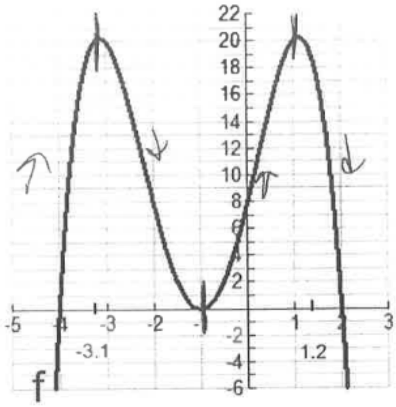


The graph of f is strictly increasing, so f' will always be positive. Since f'' is the derivative of f', it should be positive when f' is increasing and negative when f' is decreasing, which f'' is. Also, f' has a rel. max when X = 7, and f'' has an x-int when X = 7.

Lessons 3-1, 3-2, 5-3 Relating the Graphs of Part 1

Practice

1. The graph to the right is a graph of f, the original function. Use the graph to answer the following questions



   a. What intervals will the graph of f' be positive?

   $(-\infty, -3.1) \cup (-1, 1.2)$

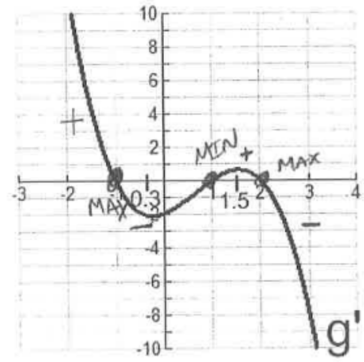
   b. What intervals will the graph of f' be negative?

   $(-3.1, -1) \cup (1.2, \infty)$

   c. What are the zeroes of f'?

   $x = -3.1, -1, 1.2$

2. The graph to the right is a graph of g', the derivative of g. Use the graph to answer the following questions.



   a. On what interval(s) is the graph of g increasing?

   $(-\infty, -1) \cup (1, 2)$

   b. On what interval(s) is the graph of g decreasing?

   $(-1, 1) \cup (2, \infty)$

   c. When does g have a local maximum?

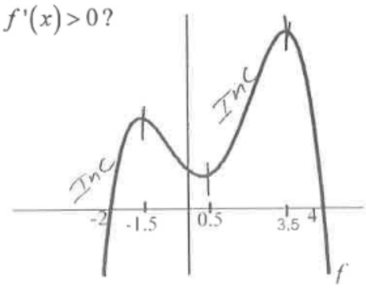
   $x = -1, x = 2$

   Local minimum?

   $x = 1$

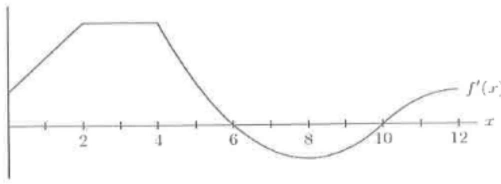
3. To the right is the graph of $f(x)$. On what intervals is $f'(x) > 0$?

$(-\infty, -1.5) \cup (.5, 3.5)$



Lessons 3-1, 3-2, 5-3 Relating the Graphs of Part 1

4. Consider the graph below of the derivative of f.



For each of the following, circle ALL correct answers.

a. $f(x)$ has a relative minimum when $x =$

0 2 4 6 8 10 12

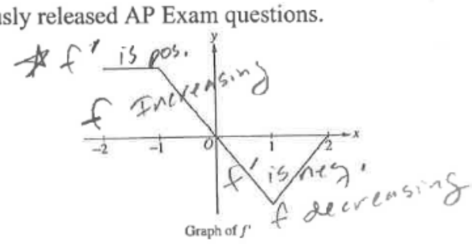
b. On which of the following intervals is $f(x)$ increasing?

(0, 2) (2, 4) (4, 6) (6, 8) (8, 10) (10, 12)

Stop, Drop, and AP!

Complete the following previously released AP Exam questions.

5.



The graph of f', the derivative of the function f, is shown above. Which of the following statements is true about f ?

(A) f is decreasing for $-1 \le x \le 1$. ✗

(B) f is increasing for $-2 \le x \le 0$. ✓

(C) f is increasing for $1 \le x \le 2$. ✗

(D) f has a local minimum at $x = 0$. ✗ incr → Dec so local max

(E) f is not differentiable at $x = -1$ and $x = 1$. ✗ → It is diff b/c the derivative is shown.

6.

x	−4	−3	−2	−1	0	1	2	3	4
$g'(x)$	2	3	0	−3	−2	−1	0	3	2

→ x-int → x-int

The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?

(A) $-2 \le x \le 2$ only

(B) $-1 \le x \le 1$ only

(C) $x \ge -2$

(D) $x \ge 2$ only

(E) $x \le -2$ or $x \ge 2$

★ g is decreasing when g' is negative.

→ Since g has exactly 2 zeros, $g' < 0$ $-2 < x \le 1$ and $1 \le x < 2$

★ technically $-2 < x < 2$

Lessons 3-1, 3-2, 5-3 Relating the Graphs of Part 1

AP Calculus AB
Lessons 3-1, 3-2, 5-3 Part 2 Warm-up

Name Heint
Date 2016

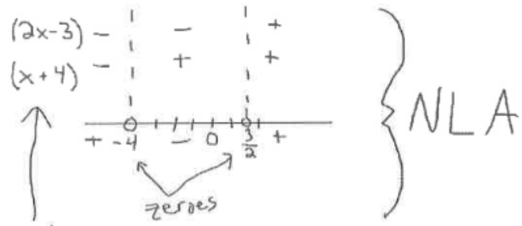
Example: Find the derivative of the given function. Use Number Line Analysis (NLA) to determine on what intervals the **derivative** is positive.

$$f(x)=\frac{2}{3}x^3+\frac{5}{2}x^2-12x+6$$

$f'(x)=2x^2+5x-12 \leftarrow$ Derivative (power rule)

$2x^2+5x-12>0 \leftarrow$ positive

$(2x-3)(x+4)>0 \leftarrow$ factor



$\boxed{(-\infty,4)\cup(\frac{3}{2},\infty)}$

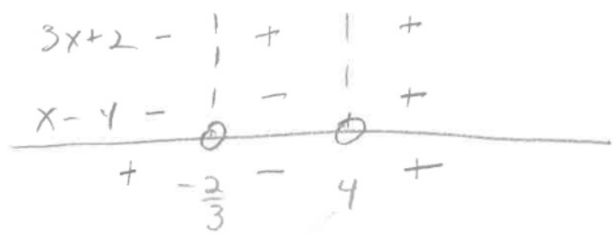
Now you try: Find the derivative of the given function. Use NLA to determine on what intervals the **derivative** is positive and on which intervals it is negative.

$$g(x)=x^3-5x^2-8x+2$$

$g'(x)=3x^2-10x-8$

$g'(x)=(3x+2)(x-4)$

$x=-\frac{2}{3} \quad x=4$



$g'(x)>0$ at $(-\infty,\frac{2}{3})\cup(4,\infty)$ $g'(x)<0$ at $(-\frac{2}{3},4)$

***Challenge:** Without graphing in your calculator, determine the coordinates of the relative max. How do you know this is the relative max?*

At $x=\frac{2}{3}$ g' turns from pos. to neg so this is the rel. max.

$g(-\frac{2}{3})=(-\frac{2}{3})^3-5(-\frac{2}{3})^2-8(-\frac{2}{3})+2=130/27=4.\overline{814}$

$(-\frac{2}{3},4.\overline{814})$ rel. max.